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Optimization of Patrol Manpower Allocation Using Goal Programming Approach -A Case Study

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ABSTRACT
One of the most difficult tasks of the patrol administrators is allocation of manpower; i.e. determining the. most effective level of operational manpower for patrol tasks. Typically, administrators resolve the allocation problem by relying on prior statistical data and by employing subjective analysis. In general, only limited systematic analyses have been applied to the problem. This thesis presents a non linear goal programming model for allocating patrolmen to road segments within a patrol region. The model is demonstrated via a case example of the East section of Visakhapatnam. The results of the model are valuable to the patrol administrator for considering departmental goals and priority structure, in addition to available historical data, in the assignment of patrol manpower for a given urban area.
Keywords - Goals, Patrolmen Allocation, Non Linear Goal Programming

I. Introduction

The primary functions of the patrol department are to enforce laws regulating the operation of vehicles on unincorporated roadways, to investigate accidents, assist motorists, provide emergency services, direct traffic, and provide limited criminal law enforcement assistance. For the most part there are only slight variations in these duties among the various patrol organizations within the reach.

In order to be successful in their primary task of traffic law enforcement, the patrol attempts to obtain voluntary compliance with traffic rules and regulations by all motorists and pedestrians. Since many drivers are inclined to violate laws unless strong deterrents are developed and implemented, enforcement needs to be intelligently, directly, and efficiently applied.

II. Literature Review

- Raja Jayaraman et al. (2015) stated that sustainable development is an important and strategic priority for global nations which requires simultaneously satisfying multiple conflicting objectives involving social, economic, energy, and environmental constraints. He proposed a goal programming model that integrates efficient allocation of labour resources to achieve sustainability objectives relating to economic, energy and environmental goals of the United Arab Emirates by the year 2030.
- Aleksander Aristovnik, Janko Seljak, Jernej Mencinger (2014) have discussed the methodology to allocate the patrol manpower according to the efficiency scores obtained by applying it in a given local domain or area. A three-stage Data Envelopment Analysis (DEA) technique is presented and then applied to measure the relative efficiency of police-work-related data for selected police units at the local level.
- Taylor B. W. et al. (1985) described a modeling approach that can be used to deploy state highway patrol cars to the road segments comprising a highway patrol district. Specifically, an integer nonlinear goal programming model was used and demonstrated within the context of a hypothetical case example. The model reflected the fact that the relationships between unit deployment and performance measures are often non linear as additional patrol units are assigned to a road segment, performance will increase but at a decreasing rate.

III. Goal Programming

The theory of Goal Programming was well defined by Nobel Laureate Herbert A. Simon's theory of "Satisfaction". He stated that today's manager does not attempt to optimize, instead, the manager (decision maker) tries to satisfy. It is a powerful tool that draws upon the highly developed and tested technique of linear programming but at the same provides a simultaneous solution to a complex system of competing objectives. A major advantage of goal programming over other techniques in dealing with real-world decision problems is that it reflects the way manager actually makes decision. In other words goal programming allows the decision maker to incorporate the environmental, organizational and managerial considerations into the model through goal levels and priorities. Goal programming is an extension of linear programming and provides a simultaneous solution to a complex system of competing objectives. This concept was originally developed by Charnes and Cooper (1961), and its methodology has been further refined by Ijiri (1965), Lee (1972) and others. In goal programming, instead of trying to maximize or minimize the objective function directly, the deviation between goals and what can be achieved within the given set of constraints are to be minimized

The basic approach of goal programming is to establish a specific numeric goal for each of the objectives, formulate an objective function for each objective, and then seek a solution that minimizes the (weighted) sum of deviations of these objective functions from their respective goals.

3.1 Problem formulation

The Procedure for formulating a goal programming problem involves the following crucial steps:

1.) Determining the Decision Variables.
2.) Formulation of the Objective Function.
3.) Absolute Objectives
4.) Formulation of Achievement function
5.) Priority Structure

3.1.1 Determining the Decision Variables

The decision variables are those factors over which the decision-maker has control, for this reason they are sometimes known as the 'control' variables.

3.1.2 Formulation of the objective function

In this context, the goals or objectives are designated by G. Each objective function in turn must be expressed as a function of the decision variables i.e.,

$$G_i = f_i(x)\begin{cases} \geq b_i \\ = b_i \\ \leq b_i \end{cases}$$

Where $f_i(x)$ is the function of decision variables associated with the i^{th} objectives and b_i is the associated right-hand side value and reflects the value that $f_i(x)$ must satisfy exceed or be less than. In addition, every objective is accompanied, on the left-hand side, by a negative and a positive deviation variable n_i and p_i respectively. The value of p_i for any choice of x, reflects the positive deviation. Consequently, each objective will be of the following final form.

$$f_i(x) + n_i - p_i = b_i \qquad i=1, 2,....,m.$$

3.1.3 Absolute Objectives

If for any objective, either p_i or n_i must equal zero for the solution to be implementable, such an objective is denoted as an absolute i.e., One that must be satisfied. This can be accomplished by assigning top priority p, to those absolute objectives.

3.1.4 Formulation of the Achievement Function

The above discussion for formulating the objection function is considered and followed by the formulation of the achievement function. Thus, the achievement function will then take on the form shown below:

Minimise

$$\overline{a} = [g_1(\overline{n}, \overline{p}), g_2(\overline{n}, \overline{p}), ...g_k(\overline{n}, \overline{p})]$$

Or alternatively

Minimise $\overline{a} = (a_1, a_2, ...a_k)$, Where $a_k = g_k(\overline{n}, \overline{p})$ and $g_k(\overline{n}, \overline{p})$ is a linear function of the deviation variable

3.1.5 Priority Structure

It is assumed that objectives may be arranged according to ordered sets having been assigned such priorities that $p_k >> p_{k+1}$. These preemptive priorities do not represent numbers and should never be replaced with numbers. For solving any Goal Programming model firstly, it involves achieving highest priority goal, before any goals of the lower priority are considered. Once the highest priority goal is attained to the fullest extent possible, the Goal Programming model proceeds to find a satisfactory level to next highest priority goal, and so on. However, it is not always possible to achieve every goal to the extent desired by the decision maker. But the advantage of linear Goal Programming over the ordinary Linear Programming is that, it seeks, within the given set of constraints, to minimize the deviation from the established goals targets, while the ordinary Linear Programming seeks to minimize or maximize certain goal directly.

IV. Model Formulation

A Non-linear Goal Programming (NLGP) model was developed for allocation of patrolmen with Multiple goals. The goals include, Total patrolmen allocation, Total budget allocation, Minimum shift requirement reduction, Estimated accident reduction, Accident reduction rate in high accident areas. Most of the road segments in urban areas have high traffic density due to the limited road area. . All the Road segments considered are Intra-city road segments. In this context, the total working hours has been divided into three shifts and the shifts are given as

First Shift: 7:00 am to 3:00 pm
Second Shift: 3:00 pm to 11:00 pm
Third Shift: 11:00 pm to 7:00 am

4.1 Problem Formulation

The formulation of the model of the problem involves indentifying the goals and creating a priority structure to build the model. Hence, the formulation of the problem includes

1. Total patrolmen goal
2. Total budget goal
3. Minimum shift requirement goal
4. Estimated accident reduction goal
5. Accident rate reduction in high accident prone road segments goal
6. Priority Structure

4.1.1 Total Patrol Men Goal

For effective functioning of patrolling activities, it is desired to allocate the patrolmen properly. Since daily the available patrolmen are limited, it is desirable to utilize the available patrolmen for all the road segments in the city to achieve the target. Hence it is given as,

$$\sum_{i=1}^{n}\sum_{j=1}^{3}\left(X_{ij}\right)+d_1^- - d_1^+ = P \qquad \text{(i)}$$

Where P is the total number of Patrolmen available for allocation, i= Road Segments 1, 2,.......n and j= Shits 1, 2, 3.

4.1.2 Total Budget Goal

For immediate maintenance of the patrol vehicle and fuel charge. The Government, provides some amount of money to each vehicle daily. Hence it is given as:

$$\sum_{i=1}^{n}\sum_{j=1}^{3}\left(C_i X_{ij}\right)+d_2^- - d_2^+ = P \qquad \text{(ii)}$$

Where P is the maximum budget available. C_i is the variable cost (□) per patrol vehicle per day with respect to the corresponding road segment *i*

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4.1.3 Minimum Shift Requirement Goal

All the segments have different traffic density and accident frequency as mentioned before. Hence, In order to provide emergency services, a minimum number of patrolmen required for a.) Road Segments which are linked together and b.) Individual Road Segments. The minimum no. of patrolmen required are estimated according to the traffic density and accident frequency of each road segment. Hence, the goal equations are given as follows:

$$\text{a.)}\sum_{j=1}^{3}(X_{i\,j} + X_{i'j} + X_{i''j} + ...) + d_3^- - d_3^+ = M \qquad \text{(iii)}$$

$$\text{b.)}\sum_{j=1}^{3}(X_{ij}) + d_4^- - d_4^+ = N \qquad \text{(iv)}$$

Where a) corresponds to the goal equation of a junction or road segments that are linked together and b) corresponds to the individual road segments. M and N represent the respective requirement of police patrolmen for the given junction and individual road segment.

4.1.4 Estimated Total Accident Reduction Goal

The reduction in accident rate (measured as accidents per Km travelled) depends on the number of patrolmen assigned to the road segment during the shift j. The accident rate for a particular road segment will be reduced with the proper of allocation patrolmen. So, the goal equation will be of the form:

$$\frac{1}{L}\left[\sum_{i=1}^{n}\sum_{j=1}^{3}\left(a_{ij} - l_i\left(\frac{b_{ij}}{X_{ij}}\right)\right)\right] + d_n^- - d_n^+ = Q_n \qquad \text{(v)}$$

Where, Q_n is the estimated reduction in the number of accidents per Km.

l_i is the length of the respective road segment *i*. *L* is the total length of all the road segments.

The values of parameters a_{ij} and b_{ij} can be estimated by defining the relationship for two point estimates i.e., the number of patrolmen per shift and the number of accidents per Km and solving the obtained equations simultaneously.

4.1.5 High Accident Prone Road Segments Goal

In addition to achieve the total accident rate reduction target of all the road segments as mentioned before, it is required to achieve the individual accident rate reduction target of each road segment. The goal equations are of the form

$$\sum_{j=1}^{3}\left(a_{ij} - \frac{b_{ij}}{X_{ij}}\right) + d_n^- - d_n^+ = g_n \qquad \text{(vi)}$$

where *i*= 1, 2,....n. and g_n is the corresponding accident reduction rate of road segments.

4.1.6 Priority Structure

Hence, to carry out the optimization of patrolmen allocation, the following goals have been considered with respective priorities:

1) Estimated Total accident reduction and Accident reduction rates in high accident prone road segments. (P_1)
2) Total number of Patrolmen and the minimum shift requirement.(P_2)
3) Total Budget expenditure per day (P_3)

In this decision making situation, four cases have been considered accordingly with respect to the goal considered as above. This allocates the patrolmen in the given road segments and in their corresponding shifts. A non linear goal programming method is adopted to carry out the optimization. The formulation of the above four cases is given below:

Case 1: All the priorities (P_1, P_2 & P_3) are considered altogether simultaneously and the achievement function is given as:

Minimise $V(d) = \sum_{i=1}^{n} \left(d_i^+\right)$ subject to (i to vi)

Case 2: Only P_1 is considered in this case. Hence the function will be:

Minimise $V(d) = \sum_{i=m}^{n} \left(d_i^+\right)$ subject to (i to vi)

Case 3: Only P_2 is considered in this case. Hence the function will be:

Minimise $V(d) = \sum_{\underbrace{i=k}_{i \neq 2}}^{m} \left(d_i^+\right)$ subject to (i to vi)

Case 4: Only P_3 is considered in this case. Hence the function will be:

Minimise $V(d) = d_2^+$
subject to (i to vi)

The above Non- linear Goal Programming (NLGP) models (case 1, case 2, case 3 and case 4) are solved by using Lingo 9.0 Solver.

V. Case Study

The considered region for the case study which is the East Section of Visakhapatnam which is categorized in two sub zones. They are namely, East-I zone and East-II zone respectively. The East-I zone is under the jurisdiction of the III Town Police Station and East-II zone is under the jurisdiction of II Town Police station respectively. The roads and Junctions considered are those with high traffic density and all are intra road segments . These details can be viewed from the map enclosed in Fig 5.1, Fig 5.2 and Fig 5.3.

5.1 Problem Formulation

Firstly, the number of patrolmen to be allocated is considered to be the decision variable which is denoted by X_{ij} , such that it gives the number of patrolmen assigned to the road segment i within the corresponding shift j.

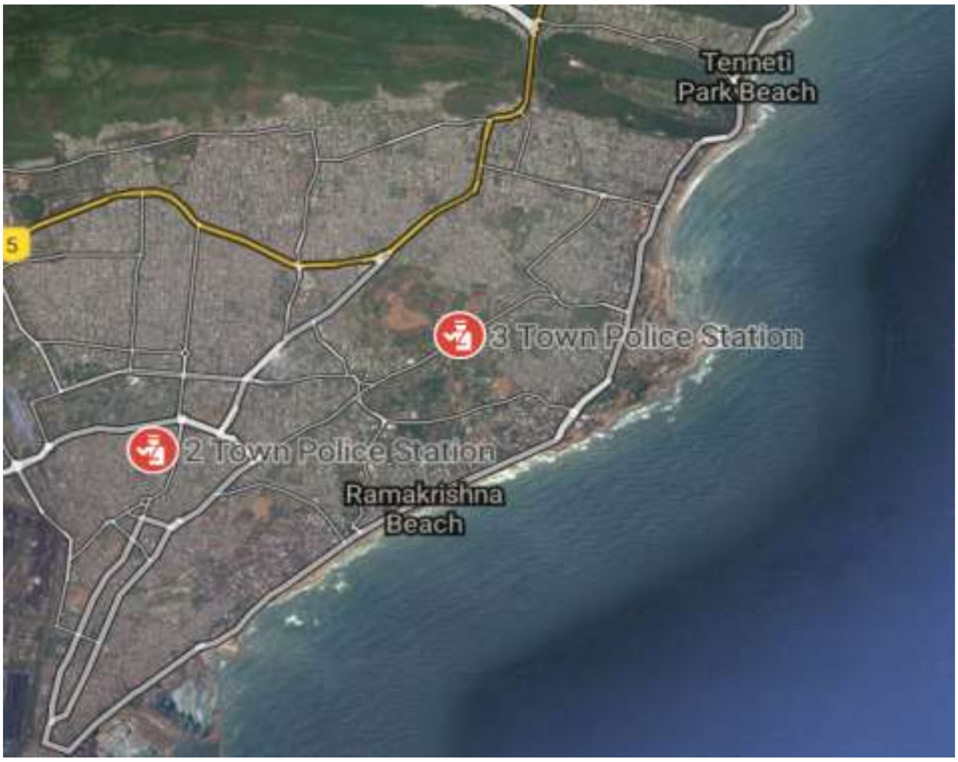

Fig 5.1 Visakhapatnam East Section Limits

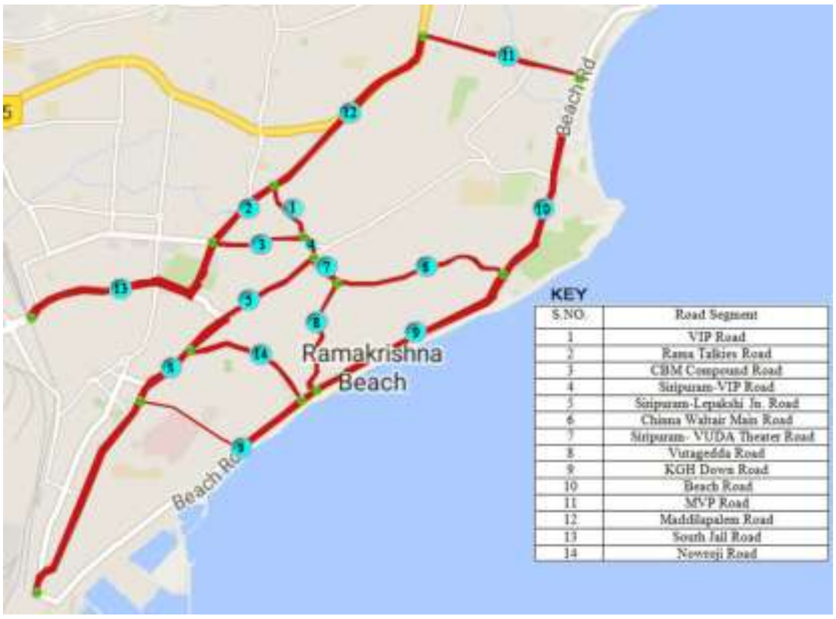

Fig 5.2 Considered Road Segments

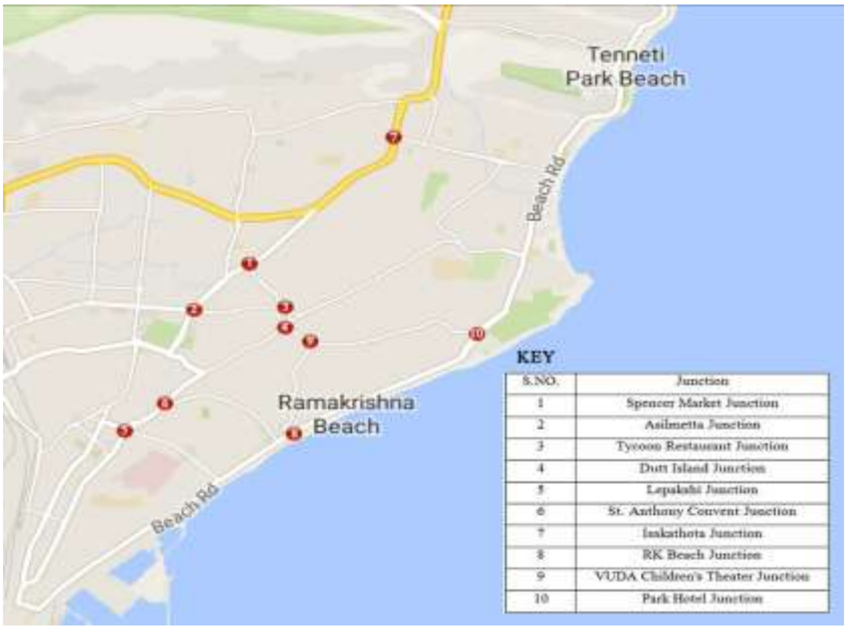

Fig 5.3 Considered Junctions

Secondly, the Goals have been formulated as per the discussion in the section of model formulation. These goals have been considered as constraints to which the objective function will be subject to.

5.1.1 Total Patrol Men Goal

This goal has been formulated as per the data given in table 5.1:

Table 5.1 Police Statistics of II Town and III Town Police Stations as on 18-07-2015

Name of the Police Station	Actual Strength					Total
	CI	SI	ASI	HC	PC	
Traffic North	1	-	-	-	-	1
II Town	-	1	4	6	50	61
III Town	-	2	4	7	40	53
Total	1	3	8	13	90	115

Based on the above Statistic obtained the total number of patrolmen (PC) required is 90. Hence the goal is formulated as

$$\sum_{i=1}^{14}\sum_{j=1}^{3}\left(X_{ij}\right)+d_1^- - d_1^+ = 90$$

5.1.2 Total Budget Goal

Based on the data collected from the authorities, the traffic department expends □1100 per day to patrol the considered section of Visakhapatnam. The total length of all the considered road segments is 29.16 km and the variable cost per km is □ 5.00, based on which the corresponding cost per km table has been formulated as below as shown in table 5.2:

Table 5.2 Variable cost for each Road Segment

Road Segment (i)	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Length(Km)	0.6	0.9	0.9	0.26	4.5	3.8	0.3	1.3	3.5	4.5	1.6	2.2	3.8	1
Cost (C_i)	3	4.5	4.5	1.3	22.5	19	1.5	6.5	17.5	22.5	8	11	19	5

The Hence the corresponding goal equation is given as:

$$\sum_{i=1}^{14}\sum_{j=1}^{3}\left(C_i X_{ij}\right)+d_2^- - d_2^+ = 1100$$

5.1.3 Minimum Shift Requirement Goal

All the segments have different traffic density and accident frequency as mentioned before. Hence, In order to provide emergency services, a minimum number of patrolmen is assigned as shown in the table above. It is required for a) Road segments which are linked together or Junctions and b) Each road segment. Based on this corresponding data has been collected and formulated as shown in Table 5.3.

Table 5.3 Minimum Shift Requirement

Road Segment	Shifts			Total
	1st Shift (8 hrs)	2nd Shift (8 hrs)	3rd Shift (8 hrs)	
1	2	2	1	5
2	2	1	2	5
3	1	2	2	5
4	2	1	1	4
5	3	3	2	8
6	3	2	2	7
7	1	1	1	3
8	3	1	1	5
9	4	2	2	8
10	2	3	2	7
11	2	2	2	6
12	4	4	2	10
13	5	5	2	12
14	3	1	1	5
Total				90

Based on the above formulated table the following goal equations have been formulated as discussed in earlier section, for both Junctions and individual road segments:

Junctions

1) $\sum_{j=1}^{3}(X_{1j}+X_{2j}+X_{12j})+d_3^- - d_3^+ = 19$

2) $\sum_{j=1}^{3}(X_{2j}+X_{3j}+X_{13j})+d_4^- - d_4^+ = 21$

3) $\sum_{j=1}^{3}(X_{1j}+X_{3j}+X_{4j})+d_5^- - d_5^+ = 13$

4) $\sum_{j=1}^{3}(X_{4j}+X_{5j}+X_{7j})+d_6^- - d_6^+ = 14$

5) $\sum_{j=1}^{3}\left(X_{9j}+X_{5j}\right)+d_7^- - d_7^+ = 14$

6) $\sum_{j=1}^{3}(X_{5j}+X_{14j})+d_8^- - d_8^+ = 12$

7) $\sum_{j=1}^{3}\left(X_{12j}+X_{11j}\right)+d_9^- - d_9^+ = 14$

8) $\sum_{j=1}^{3}(X_{8j}+X_{9j})+d_{10}^- - d_{10}^+ = 12$

9) $\sum_{j=1}^{3}(X_{6j}+X_{7j}+X_{8j})+d_{11}^- - d_{11}^+ = 13$

10) $\sum_{j=1}^{3}(X_{6j}+X_{9j}+X_{10j})+d_{12}^- - d_{12}^+ = 20$

Individual Road Segments

1) $\sum_{j=1}^{3}(X_{1j})+d_{13}^- - d_{13}^+ = 4$

2) $\sum_{j=1}^{3}(X_{2j})+d_{14}^- - d_{14}^+ = 4$

3) $\sum_{j=1}^{3}(X_{3j})+d_{15}^- - d_{15}^+ = 4$

4) $\sum_{j=1}^{3}(X_{4j})+d_{16}^- - d_{16}^+ = 4$

5) $\sum_{j=1}^{3}(X_{5j})+d_{17}^- - d_{17}^+ = 7$

6) $\sum_{j=1}^{3}(X_{6j})+d_{18}^- - d_{18}^+ = 7$

7) $\sum_{j=1}^{3}(X_{7j})+d_{19}^- - d_{19}^+ = 3$

8) $\sum_{j=1}^{3}(X_{8j})+d_{20}^- - d_{20}^+ = 5$

9) $\sum_{j=1}^{3}(X_{9j})+d_{21}^- - d_{21}^+ = 7$

10) $\sum_{j=1}^{3}(X_{10j})+d_{22}^- - d_{22}^+ = 7$

11) $\sum_{j=1}^{3}(X_{11j})+d_{23}^- - d_{23}^+ = 5$

12) $\sum_{j=1}^{3}(X_{12j})+d_{24}^- - d_{24}^+ = 9$

13) $\sum_{j=1}^{3}(X_{13j})+d_{25}^- - d_{25}^+ = 11$

14) $\sum_{j=1}^{3}(X_{14j})+d_{26}^- - d_{26}^+ = 6$

5.1.4 Estimated Total Accident Reduction Goal

The formulation for this goal can be referred with (iv), as discussed in the model formulation section. The corresponding data is taken from the accident statistics of 2013-2014 which is formulated as shown in table 5.4

Table 5.4 Accident Statistics for the years 2013-2014

Road segment	No. of accidents								Length	Rate (2014)	Target for 2015
	2013				2014				(Km)		(rate*0.90)
Shifts	I	II	III	Total	I	II	III	Total			
1	5	3	3	11	6	4	2	12	0.6	20	18
2	9	4	2	15	11	3	3	17	0.9	18.88	16.992
3	8	5	3	16	9	6	2	17	0.9	18.88	16.992
4	4	2	2	8	3	2	1	6	0.26	15.38	13.84
5	16	14	8	38	20	12	6	38	4.5	8.44	7.39
6	12	10	3	25	15	6	4	25	3.8	6.57	5.92
7	1	2	1	4	1	1	1	3	0.3	10	9
8	5	5	1	11	3	2	2	7	1.3	5.38	4.84
9	9	6	4	19	8	9	3	20	3.5	5.71	5.14
10	12	7	4	23	13	7	5	25	4.5	5.55	5
11	9	8	5	22	11	6	3	20	1.6	12.5	11.25
12	18	14	5	37	15	16	7	38	2.2	17.27	15.54
13	6	5	1	12	7	6	2	15	3.8	3.94	3.54
14	3	3	2	8	5	2	2	9	1	9	8.1
Total				248				252	29.16	7.16	6.45

Hence proceeding further using the above data, and evaluating the parametric values of a_{ij} and b_{ij}, we obtain the their corresponding values as shown in table 5.5

Table 5.5 Parameter values of a_{ij} and b_{ij}

Road segment	shift	no. of accident/km		no. of patrolmen		a_{ij}	b_{ij}
		2013	2014	2013	2014		
1	1	8.33	10	2	3	13.34	9.82
	2	5	6.66	1	3	7.47	2.47
	3	5	3.33	2	2	1.77	-9.76
2	1	10	12.22	3	1	8.9	-3.31
	2	4.44	3.33	2	1	5.55	2.22
	3	2.22	3.33	1	2	4.44	2.22
3	1	8.88	10	3	1	8.32	-1.67
	2	5.55	6.66	1	3	7.2	1.65
	3	3.33	2.22	3	2	5.48	6.52
4	1	15.38	11.53	1	3	9.63	-5.74
	2	7.69	7.69	2	1	7.69	0
	3	7.69	3.84	2	1	11.54	7.7
5	1	3.55	4.44	4	2	3.84	-1.18
	2	3.11	2.66	1	4	2.75	-0.36
	3	1.77	1.33	2	4	0.89	-1.76
6	1	6.66	8.33	4	2	2.367	-3.16
	2	5.55	3.33	1	3	1.04	-1.58
	3	1.66	2.22	2	2	1.56	1.54
7	1	3.33	3.33	2	2	3.33	0
	2	6.66	3.33	1	1	9.99	6.66
	3	3.33	3.33	1	2	3.33	0
8	1	3.84	2.30	4	3	8.64	19.25
	2	3.84	1.53	3	1	4.97	3.44
	3	0.76	1.53	1	3	1.9	1.13
9	1	2.57	2.28	5	3	3.01	2.23
	2	1.71	2.57	2	4	3.43	3.44
	3	1.14	0.85	3	2	1.68	1.66
10	1	2.66	2.88	1	2	3.1	0.44
	2	1.55	1.55	2	4	1.55	0
	3	0.88	1.11	2	1	0.65	-0.46
11	1	5.62	6.87	1	3	7.49	1.86
	2	5	3.75	2	3	1.32	-7.35
	3	3.12	1.87	1	2	0.62	-2.5
12	1	8.18	6.81	3	5	4.7	-10.53
	2	6.36	7.27	4	4	2.72	-18.2
	3	2.27	3.18	2	1	1.36	-1.82
13	1	1.57	1.84	6	4	1.03	-3.24
	2	1.31	1.57	5	5	2.61	5.2
	3	0.26	0.52	2	3	1.02	1.53
14	1	3	5	4	2	1	-8
	2	3	2	1	1	1	-2
	3	2	2	2	1	2	0

By substituting these values in the given model equation (v), we obtain our goal equation for this goal equation as:

$$\frac{1}{29.16}\left[\begin{array}{l} 176.23-0.6\left(\frac{9.82}{X_{11}}+\frac{2.47}{X_{12}}-\frac{9.76}{X_{13}}\right)-0.9\left(\frac{-3.31}{X_{21}}+\frac{2.22}{X_{22}}+\frac{2.22}{X_{23}}\right) \\ -0.9\left(\frac{-1.67}{X_{31}}+\frac{1.65}{X_{32}}+\frac{6.52}{X_{33}}\right)-0.26\left(\frac{-5.74}{X_{41}}+\frac{7.7}{X_{43}}\right) \\ -4.5\left(\frac{-1.18}{X_{51}}-\frac{0.36}{X_{52}}-\frac{1.76}{X_{53}}\right)-3.8\left(\frac{-3.16}{X_{61}}-\frac{1.58}{X_{62}}+\frac{1.54}{X_{63}}\right) \\ -0.3\left(\frac{6.66}{X_{72}}\right)-1.3\left(\frac{19.25}{X_{81}}-\frac{3.44}{X_{82}}+\frac{1.13}{X_{83}}\right) \\ -3.5\left(\frac{2.23}{X_{91}}+\frac{3.44}{X_{92}}+\frac{1.66}{X_{93}}\right)-4.5\left(\frac{0.44}{X_{101}}-\frac{0.46}{X_{103}}\right) \\ -1.6\left(\frac{1.86}{X_{111}}-\frac{7.35}{X_{112}}-\frac{2.5}{X_{113}}\right)-2.2\left(\frac{-10.53}{X_{121}}-\frac{18.2}{X_{122}}-\frac{18.2}{X_{123}}\right) \\ -3.8\left(\frac{-3.24}{X_{131}}+\frac{5.2}{X_{132}}+\frac{1.53}{X_{133}}\right)-\left(\frac{-8}{X_{141}}-\frac{2}{X_{142}}\right) \end{array}\right]$$

$$+d_{27}^{-}-d_{27}^{+}=6.45$$

5.1.5 Accident Rate Reduction in High Accident Prone Segments Goal

Similarly, as per the discussed model formulation (vi), for this goal and using the data given above, the corresponding parameters are substituted and the goal equations are obtained as follows:

1) $$22.58-\frac{9.82}{X_{11}}-\frac{2.47}{X_{12}}+\frac{9.76}{X_{13}}+d_{28}^{-}-d_{28}^{+}=18$$

2) $$18.89+\frac{3.31}{X_{21}}-\frac{2.22}{X_{22}}-\frac{2.22}{X_{23}}+d_{29}^{-}-d_{29}^{+}=16.992$$

3) $$21+\frac{1.67}{X_{31}}-\frac{1.65}{X_{32}}+\frac{6.52}{X_{33}}+d_{30}^{-}-d_{30}^{+}=16.992$$

4) $$28.86+\frac{5.74}{X_{41}}-\frac{7.7}{X_{43}}+d_{31}^{-}-d_{31}^{+}=13.842$$

5) $$7.48-\frac{1.18}{X_{51}}-\frac{0.36}{X_{52}}+\frac{1.76}{X_{53}}+d_{32}^{-}-d_{32}^{+}=7.6$$

6) $$4.97+\frac{3.16}{X_{61}}+\frac{1.58}{X_{62}}-\frac{1.54}{X_{63}}+d_{33}^{-}-d_{33}^{+}=5.92$$

7) $$16.65-\frac{6.66}{X_{72}}+d_{34}^{-}-d_{34}^{+}=9$$

8) $$15.51-\frac{19.25}{X_{81}}-\frac{3.44}{X_{82}}-\frac{1.13}{X_{83}}+d_{35}^{+}-d_{35}^{-}=4.84$$

9) $$8.12-\frac{2.23}{X_{91}}-\frac{3.44}{X_{92}}-\frac{1.66}{X_{93}}+d_{36}^{-}-d_{36}^{+}=5.14$$

10) $$5.3-\frac{0.44}{X_{101}}+\frac{0.46}{X_{103}}+d_{37}^{-}-d_{37}^{+}=5$$

11) $$9.435-\frac{1.86}{X_{111}}+\frac{7.35}{X_{112}}+\frac{2.5}{X_{113}}+d_{38}^{-}-d_{38}^{+}=11.25$$

12) $$8.78+\frac{10.53}{X_{121}}+\frac{18.2}{X_{122}}+\frac{1.82}{X_{123}}+d_{39}^{-}-d_{39}^{+}=15.54$$

13) $$4.66+\frac{3.24}{X_{131}}-\frac{5.27}{X_{132}}-\frac{1.53}{X_{133}}+d_{40}^{-}-d_{40}^{+}=3.546$$

14) $$4+\frac{8}{X_{141}}+\frac{2}{X_{142}}+d_{41}^{-}-d_{41}^{+}=8.1$$

5.1.6 Priority Structure

A non linear goal programming method is adopted to carry out the optimization. The formulation of the above four cases is given below:

Case 1: All the priorities (P_1, P_2 & P_3) are considered altogether simultaneously and the achievement function is given as:

Minimise $V(d)=\sum_{i=1}^{41}\left(d_i^{+}\right)$

subject to all the constraints in 5.1.1 to 5.1.5.

Case 2: Only P_1 is considered in this case. Hence the function will be:

Minimize $V(d)=\sum_{i=27}^{41}\left(d_i^{+}\right)$

subject to all the constraints in 5.1.1 to 5.1.5

Case 3: Only P_2 is considered in this case. Hence the function will be:

Minimise $V(d)=\underbrace{\sum_{i=1}^{26}}_{i\neq 2}\left(d_i^{+}\right)$

subject to all the constraints in 5.1.1 to 5.1.5.

Case 4: Only P_3 is considered in this case. Hence the function will be:

Minimise $V(d)=d_2^{+}$

subject to all the constraints in 5.1.1 to 5.1.5.

The above Non- linear Goal Programming (NLGP) models (case 1, case 2, case 3and case 4) are solved by using Lingo 9.0 Solver.

VI. Result and Discussions

Based on the formulated model for the case study and previous discussions, the NLGP models were executed in Lingo 9.0 Solver to obtain the corresponding results for the allocation of patrolmen for the considered four cases individually. This model uses an integer non linear goal programming approach to solve the need for reducing traffic violations and accidents by optimum allocation of patrolmen in various road segments in different shifts.

This type or model can be utilized to the help management of traffic police department in Visakhapatnam city, in order to reduce manual effort and to achieve optimal allocation of existing patrolmen depending upon the given goal constraints and priorities. The obtained results have tabulated accordingly to formulate individual tables which have been discussed in the following section. The formulated results have been categorized into two types of tables:

- Optimal Allocation
- Goal Fulfillments

6.1 Optimal Allocation

This analysis gives the number of patrolmen allocated under consideration of each case. The value obtained are integers obtained from solving the NLGP models as discussed in the previous sections of the Priority Structure of the Case Study Problem formulation. These results are formulated as shown in the table 6.1

Table 6.1 Optimal Allocation of Patrolmen for all the considered four cases

Road Segment	Shifts	Case 1		Case 2		Case 3		Case4	
1	1	1		1		1		2	
	2	1		1		2		1	
	3	1	3	1	3	1	4	1	4
2	1	2		5		2		2	
	2	1		1		1		3	
	3	1	4	1	7	1	4	1	6
3	1	1		1		2		1	
	2	1		1		1		1	
	3	1	3	1	3	1	4	1	3
4	1	2		5		1		5	
	2	1		1		2		1	
	3	1	4	1	7	1	4	1	7
5	1	4		5		5		5	
	2	2		5		1		1	
	3	1	7	1	11	1	7	1	7
6	1	5		5		3		3	
	2	1		1		1		2	
	3	1	7	1	7	1	5	1	6
7	1	1		1		1		1	
	2	1		1		1		1	
	3	1	3	1	3	1	3	1	3
8	1	1		1		2		5	
	2	1		1		1		1	
	3	1	3	1	3	1	4	1	7
9	1	3		1		5		5	
	2	3		1		1		1	
	3	1	7	1	3	1	7	1	7
10	1	1		1		3		1	
	2	3		1		1		1	
	3	1	5	1	3	1	5	1	3
11	1	1		1		2		3	
	2	3		5		1		3	
	3	1	5	1	7	1	4	1	7
12	1	3		5		5		5	
	2	5		5		2		1	
	3	1	9	1	11	1	8	1	7
13	1	5		1		5		3	
	2	5		1		5		2	
	3	1	11	1	3	1	11	1	6
14	1	3		3		3		5	
	2	1		2		1		1	
	3	1	5	1	6	1	5	1	7
Total		76		77		75		80	

The results in the table 6.1 are the possible allocations of' patrolmen to the road segments with respect to the three shifts obtained after solving the NLGP model for all the considered four cases. Similarly, results are also obtained pertaining to the goals considered. These results have been formulated as Goal Fulfillment tables. These tables give analysis of the goal fulfillments for each of the four cases considered individually.

6.2 Goal Fulfillments

The Goal Fulfillment analysis tables are those tables which give the analysis of how far the considered goals for the given model have been fulfilled in percentiles. These tables have been further described as below:

- Firstly, There are four goal fulfillment analysis tables for the four considered cases each followed by detailed discussion of the results that are described in the individual tables.
- Each individual table consists of the goals considered, their aspiration and solution values, under and over deviations, and goal fulfillment percentages respectively.
- The following goals have been described in the tables:
  - Total Patrolmen Goal
  - Total Budget Goal
  - Minimum Shift Requirement Goal which consists of allocation for both Road Segments and Junctions.
  - Estimated Total accident Reduction Goal (given as "Est. Tot. Acc. Red." in the goal fulfillment tables)
  - High Accident Prone Road Segments

Deviations consists of under and over deviations which are denoted by d_n^- and d_n^+ respectively. These are obtained from the difference between the aspiration and the solution values. Correspondingly, the goal fulfillment percentages are calculated for each goal individually using the obtained under or over deviations respectively.

6.2.1 Goal Fulfillment Analysis for Case 1

Table 6.2 Goal Fulfillment table for case 1

S.NO	Goal Description	Aspiration Value	Solution Value	Deviations		Goal Fulfillment (%)
				d_n^-	d_n^+	
1	Total Patrolmen	90	76	14	–	Achieved
2	Total Budget (₹)	1100	968.2	131.8	-	Achieved
Minimum Shift Requirement						
3	Road Segment 1	4	3	1	-	Achieved
4	Road Segment 2	4	4	-	-	Achieved
5	Road Segment 3	4	3	1	-	Achieved
6	Road Segment 4	4	4	-	-	Achieved
7	Road Segment 5	7	7	-	-	Achieved
8	Road Segment 6	7	7	–	–	Achieved
9	Road Segment 7	3	3	-	-	Achieved
10	Road Segment 8	5	3	2	-	Achieved
11	Road Segment 9	7	7	-	-	Achieved
12	Road Segment 10	7	5	2	-	Achieved
13	Road Segment11	5	5	-	–	Achieved
14	Road Segment 12	9	9	-	-	Achieved
15	Road Segment 13	11	11	-	–	Achieved
16	Road Segment 14	6	5	1	–	Achieved
17	Junction 1	19	16	3	–	Achieved
18	Junction 2	21	18	3	–	Achieved
19	Junction 3	13	10	3	–	Achieved
20	Junction 4	14	14	-	-	Achieved
21	Junction 5	14	14	–	–	Achieved
22	Junction 6	12	12	-	-	Achieved
23	Junction 7	14	14	–	–	Achieved
24	Junction 8	12	10	2	-	Achieved
25	Junction 9	13	13	-	-	Achieved
26	Junction 10	20	19	-	-	Achieved
27	Est. Tot. Acc. Red.	6.45	5.45	1	–	Achieved
High Accident Prone Road Segments						
28	Road Segment 1	18	20.05	–	2.05	11.38
29	Road Segment 2	16.99	16.105	0.887	–	Achieved
30	Road Segment 3	16.99	14.5	2.49	-	Achieved
31	Road Segment 4	13.84	23.188	-	10.188	73.61
32	Road Segment 5	7.6	9.715	-	2.115	27.8
33	Road Segment 6	5.92	5.64	0.278	-	Achieved
34	Road Segment 7	9	9.99	-	0.99	11
35	Road Segment 8	4.84	-8.31	13.15	-	Achieved
36	Road Segment 9	5.14	4.57	0.57	-	Achieved
37	Road Segment 10	5	5.32	-	0.32	6.4
38	Road Segment 11	11.25	12.525	-	1.275	11.33
39	Road Segment 12	15.54	17.75	-	2.21	14.22
40	Road Segment 13	3.54	2.73	0.808	-	Achieved
41	Road Segment 14	8.1	8.66	–	0.56	6.91

The aspiration , solution values, deviations and corresponding goal fulfillment percentages of the goals considered under case 1 can be seen from the above formulated table 6.2. In case 1 the priorities P_1, P_2, and P_3 are considered altogether simultaneously. It can be seen that all the Minimum Shift Requirement goals are achieved, but there are some overachievements observed in case of Accident reduction goals for high accident prone individual road segments.

6.2.2 Goal Fulfillment Analysis for Case 2

Table 6.3 Goal Fulfillment table for case 2

S.NO	Goal Description	Aspiration Value	Solution Value	Deviations d_n^-	d_n^+	Goal Fulfillment (%)
1	Total Patrolmen	90	77	13	-	Achieved
2	Total Budget (₹)	1100	851.6	248.4	-	Achieved
	Minimum Shift Requirement					
3	Road Segment 1	4	3	-	1	Achieved
4	Road Segment 2	4	7	-	3	75
5	Road Segment 3	4	3	1	-	Achieved
6	Road Segment 4	4	7	1	3	75
7	Road Segment 5	7	11	-	4	57.14
8	Road Segment 6	7	7	-	-	Achieved
9	Road Segment 7	3	3	-	1	Achieved
10	Road Segment 8	5	3	2	-	Achieved
11	Road Segment 9	7	3	4	-	Achieved
12	Road Segment 10	7	3	4	-	Achieved
13	Road Segment 11	5	7	-	2	40
14	Road Segment 12	9	11	-	2	22.22
15	Road Segment 13	11	3	8	-	Achieved
16	Road Segment 14	6	6	-	-	Achieved
17	Junction 1	19	21	-	2	11.11
18	Junction 2	21	13	8	-	Achieved
19	Junction 3	13	13	-	-	Achieved
20	Junction 4	14	17	-	3	21.42
21	Junction 5	14	21	-	7	50
22	Junction 6	12	14	-	2	16.67
23	Junction 7	14	17	-	3	21.42
24	Junction 8	12	18	-	6	50
25	Junction 9	13	13	-	-	Achieved
26	Junction 10	20	13	7	-	Achieved
27	Est. Tot. Acc. Red.	6.45	6.375	0.075	-	Achieved
	High Accident Prone Road Segments					
28	Road Segment 1	18	20.05	-	2.05	11.38
29	Road Segment 2	16.99	15.112	1.88	-	Achieved
30	Road Segment 3	16.992	14.5	2.492	-	Achieved
31	Road Segment 4	13.84	22.308	-	8.466	61.11
32	Road Segment 5	7.6	9.54	-	1.948	25.63
33	Road Segment 6	5.92	5.64	0.278	-	Achieved
34	Road Segment 7	9	9.99	-	0.99	11
35	Road Segment 8	4.84	-8.31	13.15	-	Achieved
36	Road Segment 9	5.14	0.79	4.35	-	Achieved
37	Road Segment 10	5	5.32	-	0.32	6.4
38	Road Segment 11	11.25	11.54	-	0.295	2.62
39	Road Segment 12	15.54	16.34	-	0.806	5.18
40	Road Segment 13	3.54	1.17	2.376	-	Achieved
41	Road Segment 14	8.1	7.66	0.433	-	Achieved

The aspiration , solution values, deviations and corresponding goal fulfillment percentages of the goals considered under case 2 can be seen from the above formulated table 6.3.In case 2, only P_1 is considered individually. Unlike to that in case 1, the case 2 analysis shows that the although there are certain over deviations in Accident Reduction goal, the goal achievements are better since we considered the accident reduction priority. But in case of the Minimum Shift Requirement goal, few goals have been overachieved for certain road segments and junctions.

6.2.3 Goal Fulfillment Analysis for Case 3

Table 6.4 Goal Fulfillment table for case 3

S.NO	Goal Description	Aspiration Value	Solution Value	Deviations d_n^-	d_n^+	Goal Fulfillment (%)
1	Total Patrolmen	90	75	15	-	Achieved
2	Total Budget (₹)	1100	925.2	174.8	-	Achieved
	Minimum Shift Requirement					
3	Road Segment 1	4	4	-	-	Achieved
4	Road Segment 2	4	4	-	-	Achieved
5	Road Segment 3	4	4	-	-	Achieved
6	Road Segment 4	4	4	-	-	Achieved
7	Road Segment 5	7	7	-	-	Achieved
8	Road Segment 6	7	5	2	-	Achieved
9	Road Segment 7	3	3	-	-	Achieved
10	Road Segment 8	5	4	1	-	Achieved
11	Road Segment 9	7	7	2	-	Achieved
12	Road Segment 10	7	5	2	-	Achieved
13	Road Segment11	5	4	1	-	Achieved
14	Road Segment 12	9	8	1	-	Achieved
15	Road Segment 13	11	11	-	-	Achieved
16	Road Segment 14	6	5	1	-	Achieved
17	Junction 1	19	16	3	-	Achieved
18	Junction 2	21	19	2	-	Achieved
19	Junction 3	13	12	1	-	Achieved
20	Junction 4	14	14	-	-	Achieved
21	Junction 5	14	14	-	-	Achieved
22	Junction 6	12	12	-	-	Achieved
23	Junction 7	14	12	2	-	Achieved
24	Junction 8	12	11	1	-	Achieved
25	Junction 9	13	12	1	-	Achieved
26	Junction 10	20	17	3	-	Achieved
27	Est. Tot. Acc. Red.	6.45	4.543	1.907	-	Achieved
	High Accident Prone Road Segments					
28	Road Segment 1	18	21.285	-	3.285	18.25
29	Road Segment 2	16.99	16.105	0.887	-	Achieved
30	Road Segment 3	16.99	13.665	3.327	-	Achieved
31	Road Segment 4	13.84	23.9	-	10.86	72.68
32	Road Segment 5	7.6	9.83	-	2.23	29.3
33	Road Segment 6	5.92	6.063	-	0.143	Achieved
34	Road Segment 7	9	9.99	-	0.99	11
35	Road Segment 8	4.84	1.315	3.525	-	Achieved
36	Road Segment 9	5.14	2.574	2.566	-	Achieved
37	Road Segment 10	5	5.6133	-	0.6133	12.26
38	Road Segment 11	11.25	18.355	-	7.105	63.15
39	Road Segment 12	15.54	21.806	-	6.266	40.32
40	Road Segment 13	3.54	2.738	0.808	-	Achieved
41	Road Segment 14	8.1	8.67	-	0.57	Achieved

The aspiration , solution values, deviations and corresponding goal fulfillment percentages of the goals considered under case 3 can be seen from the above formulated table 6.4. In case 3, only P_2 is considered individually. Here it can be observed that under Minimum Shift Requirement goal priority consideration, all the corresponding goals of the considered priority are achieved. But in case of Accident reduction goal, the analysis shows some certain over deviations for few road segments.

6.2.4 Goal Fulfillment Analysis for Case 4

Table 6.5 Goal Fulfillment table for case 4

S.NO	Goal Description	Aspiration Value	Solution Value	Deviations		Goal Fulfillment (%)
				d_n^-	d_n^+	
1	Total Patrolmen	90	80	10	-	Achieved
2	Total Budget (₹)	1100	855.1	244.9	-	Achieved
Minimum Shift Requirement						
3	Road Segment 1	4	4	-	-	Achieved
4	Road Segment 2	4	6	-	2	50
5	Road Segment 3	4	3	1	-	Achieved
6	Road Segment 4	4	7	-	3	Achieved
7	Road Segment 5	7	7	-	-	Achieved
8	Road Segment 6	7	6	1	-	Achieved
9	Road Segment 7	3	3	-	-	Achieved
10	Road Segment 8	5	7	-	2	40
11	Road Segment 9	7	7	-	-	Achieved
12	Road Segment 10	7	3	4	-	Achieved
13	Road Segment 11	5	7	-	2	40
14	Road Segment 12	9	7	2	-	Achieved
15	Road Segment 13	11	6	5	-	Achieved
16	Road Segment 14	6	7	-	1	16.67
17	Junction 1	19	17	2	-	Achieved
18	Junction 2	21	15	6	-	Achieved
19	Junction 3	13	14	-	1	7.69
20	Junction 4	14	17	-	3	21.4
21	Junction 5	14	14	-	-	Achieved
22	Junction 6	12	14	-	2	16.67
23	Junction 7	14	14	-	-	Achieved
24	Junction 8	12	14	-	2	16.67
25	Junction 9	13	16	-	3	23.07
26	Junction 10	20	16	4	-	Achieved
27	Est. Tot. Acc. Red.	6.45	4.23	2.22	-	Achieved
High Accident Prone Road Segments						
28	Road Segment 1	18	24.96	-	6.96	38.67
29	Road Segment 2	16.99	17.585	-	0.593	3.49
30	Road Segment 3	16.99	14.5	2.492	-	Achieved
31	Road Segment 4	13.84	22.308	-	8.46	61.12
32	Road Segment 5	7.6	9.836	-	2.236	29.42
33	Road Segment 6	5.92	5.273	0.647	-	Achieved
34	Road Segment 7	9	9.99	-	0.99	11
35	Road Segment 8	4.84	7.09	-	2.25	46.48
36	Road Segment 9	5.14	2.574	2.566	-	Achieved
37	Road Segment 10	5	5.32	-	0.32	6.4
38	Road Segment 11	11.25	13.765	-	2.515	22.35
39	Road Segment 12	15.54	25.906	-	10.366	66.7
40	Road Segment 13	3.54	1.61	1.93	-	Achieved
41	Road Segment 14	8.1	7.6	0.77	-	Achieved

The aspiration , solution values, deviations and corresponding goal fulfillment percentages of the goals considered under case 4 can be seen from the above formulated table 6.5. In case 4, only P_3 is considered individually. Here it can be observed that under the consideration Total Budget Goal Priority, Both the Minimum Shift Requirement and the Accident Reduction goals show that certain goals have been achieved while other show considerable over deviations.

It can been throughout the goal fulfillment analysis for all the considered cases 1,2, 3 and 4 the Total Patrolmen goal, Total Budget goal and the Estimated Total Accident Reduction Goal have been achieved in each considered case.

VII. Conclusions

The problem of assigning a limited number of patrolmen to an area in order to achieve maximum effectiveness has received very little attention in the law enforcement and management science literature. Previous practices of accident prediction, control chart analysis, and contact prediction for assigning patrolmen or police to various road segments or streets have been based on subjective judgments of accident and crime frequency and traffic density statistics. Since these methods are highly dependent on trial and error, they have experienced limited success.

With due time, the advent of computerized techniques came to the rescue by providing improved statistical and analytical models for solving tedious problems. Of them all, mathematical goal programming has the potential to provide better solutions and insight to many such problems. Of which, our problem of allocation of patrolmen in the given traffic domain has been solved accordingly using an NLGP model.

In an urban city like the East Section of Visakhapatnam with incremental growth in the population and changes in lifestyles, demand for more number of vehicles is increasing drastically. Heavy traffic on the roads has led to increased traffic rules violations and accidents, causing travel inconvenience and chaos within the city. It is the responsibility of the traffic police department to allocate the patrolmen in different shifts in various road segments of the city for reducing traffic rules violations, disorders and accidents.

As observed from the tables and analysis, it can be said that there are certain areas where goals like the minimum shift requirement and high accident prone road segments have been overachieved. These overachievements explain that there is high traffic density existing in those Junctions (or road segments) hence requires more allocation of patrolmen in such areas.

The methodology demonstrated how the model can be employed for an effective allocation of patrolmen. The analysis of the case study under various priorities has revealed the fulfillment of different goals. Although it is not possible to obtain a guaranteed optimal solution, this model demonstrates that a satisfactory solution can be achieved. This model has been demonstrated as a sample to show that it can also be extended to a state wide implementation of optimized patrol manpower allocation.

In this study the goal programming approach is utilized because it allows the optimization of multiple objectives while permitting an explicit consideration of the existing decision environment. The goal programming model points out that some objectives cannot be achieved under the desired conditions. Therefore, trade-offs must occur due to administrative policies or limited resources and with regard to further modifications in traffic system, policies and increase in resources, demand and population it is felt that there is a lot of scope for future work in this area.

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